# Extended Essay

## Physics

# The Physics of Water Rockets

*How does changing the initial amount of water in a water rocket affect the flight of the rocket?*

*Word Count: 3962*

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# Introduction

This extended essay aims to examine the different components of a water rockets flight. A water rocket is a small model rocket, generally made from one or two plastic pop bottles. They use pressurized air and water to generate thrust. This thrust can be explained by Newton's Third Law, which full-sized rockets operate on as well. Unlike regular rockets, however, a water rocket uses two fuel sources, water, and air. The air is used because it can be compressed, which allows it to store a large amount of energy despite having an exceedingly small mass. The air alone cannot create enough thrust to propel the rocket due to its lack of mass, so water must be used as a fuel source to generate enough force to propel the rocket upwards. Water is used because it is dense and weighs around 100 times as much as the air allowing it to create a large force downwards, which in reaction pushes the rocket upwards (NASA, 2014). There is a common misconception around what creates this force. It is believed that the water pushes down on the ground or the atmosphere, creating an equal and opposite force that propels the rocket upwards. If this were true how would a rocket work in space, where there is absolutely nothing to push off of? In order to understand where the force comes from it is better to imagine the rocket in space, where there are no external forces. If the rocket begins at rest, then the initial momentum is 0. When the water is ejected out of the rocket the rockets begin to move upwards. Now the rocket has momentum, but so did the water, which was shot out in the opposite direction. Because of the law of conservation of momentum, the rockets momentum must equal the momentum of the water (Allain, 2009). The force this process creates is called thrust, which takes place when gas or a fluid is expelled from one side of a rocket and the rocket accelerates in the opposite direction. Therefore, it is not the force of the ground/atmosphere that propels the rocket, it is the force of the air and water.

I chose this topic because rockets and space have always been extremely interesting to me. I've always been curious about how rockets work and I thought that this experiment would be an interesting and fun way to see how they work firsthand. Someday I hope to fly on a real rocket, but for now water rockets will have to do.

## Water Rockets vs. Full-Sized Rockets

For this investigation the most important difference between water rockets and full-sized rockets is the fact that water rockets contain two different propellants and therefore don't experience the same problem water rockets do regarding the balance of the two. This investigation will aim to find an optimum balance between water and air. Water is not very compressible and therefore cannot store a lot of energy. In contrast, air is very compressible and can store a lot of energy. Air is not very heavy and even when moving at a high velocity has a very small momentum. Water is heavy and when moving at the same speed will have a much higher momentum, which makes it a very good reactive mass. The air stores the energy the water will use to produce downwards momentum, which gives the water rocket the same momentum in the opposite direction, upwards.

Although full-sized rockets work in a very similar way to water rockets there are some key differences. For instance, water rockets have a much smaller propellant mass ratio. This is due to the fact that water rockets only produce thrust for a very short amount of time while full-sized rockets need to produce thrust for up to 10 minutes in order to make it into space. For reference, the propellant in a full-sized rocket makes up around 85% of its total mass (NASA, 2014). This brings up another point, because full-sized rockets spend much of their flight in space the effects of lift and drag have less influence on the rocket's trajectory. This is because aerodynamic forces like lift and drag are dependent on air density, and of course as the rocket reaches extreme heights the air density would decrease substantially (NASA, 2014). A water rocket however spends its entire flight within the earth's atmosphere, so lift and drag would have greater effects on the rockets flight. Despite the varying significance of the forces on both rockets trajectories, both rockets experience the exact same forces during a flight, thrust, gravity, lift, and drag. Because of this the water rockets flight is based off the same or very similar physics as a full-sized rocket.

The energy transformations are another thing both rockets have in common. In both rockets there is an initial storage of potential energy within the rocket. In the water rocket this comes in the form of compressed air. Pressure is a measure of how much energy is stored in a certain volume. The rocket used in this investigation was always pumped up to 275790 pascals, or 275790 $J/m^3$. This is not an accurate measurement of how high the rocket will go however, as rockets are very inefficient and don't convert all of that energy into gravitational potential energy. In a regular rocket this stored energy would come from rocket fuel, which creates thrust when combusted. After the pressure is released or the fuel is combusted the rocket will initiate take-off. This happens when the upwards force of thrust is larger in magnitude than that of weight and drag. For a water rocket this only happens for a very short amount of time as the compressed air and water are forced out of the rocket very quickly. The exact amount of time would change depending on pressure, water amount, and nozzle area. For a regular rocket there is a much higher propellant mass ratio, allowing the rocket to create thrust for a majority of its flight. This would be due to its much larger weight creating the need for constant thrust to continue movement upwards. When the rocket runs out of thrust it still continues to fly upwards, however because the rocket no longer exerts an upwards force the weight and drag cause it to decelerate. This is contrasted by a regular rocket as by the time the rocket stops exerting thrust the rocket has escaped the earth's atmosphere and no longer experiences the forces of gravity and drag to pull it back down to the ground. At the apogee of the water rockets trajectory it will have 0 kinetic energy. The kinetic energy will have transformed into gravitational potential energy. This gravitational potential energy will then again become kinetic energy as it falls towards earth. This is of course different to a full-sized rocket which usually makes it out of the earths atmosphere and doesn't fall back to earth after its apogee.

## Scope of Investigation

This purpose of this investigation is to find the optimum water to air ratio for a single stage water rocket. This will be done by conducting an experiment in which the volume of water is changed in a water rocket, and then recording the results using an altimeter. This investigation aims to explore the question **"how does changing the initial amount of water in a water rocket affect the flight of the rocket?"** This research question is worthy of investigation because it will test the validity of already existing knowledge from simulations and will clarify any misconceptions that could be derived from them. People often launch water rockets in competitions, normally aiming for a maximum altitude. With this data, it could give competitors true experimental data for an optimized fuel to air ratio. Of course, this data is limited to a pressure of 40psi, as well as rockets made from a two litre bottles, as they will have a similar mass to the rocket used in my investigation. This essay will also simultaneously explore other components of the rocket's flight and create mathematical and physical explanations of a water rockets flight.

## Apparatus

For this investigation, an apparatus capable of launching a rocket many times with consistent pressure and angle to the vertical needed to be built. **Figure 1** shows the design of launcher I used and **Figure 2** is my finished launcher. To prevent water and air leakage a bulge was created in the PVC pipe that the rocket sits on. This was done by heating the pipe and pushing both sides together. This small bulge is slightly larger than the nozzle of the rocket, meaning when pushed down by the collar, the rocket and the launcher were airtight. A foot pump was attached to the tire valve. The pump had a built in pressure gauge. The pressure gauge was vital in order to keep the pressure constant and the foot pump allowed me to transfer my kinetic energy into potential energy which becomes stored in the rocket and will be the thing that creates thrust for the rocket. The energy expended while forcing the air into the rocket chamber is equal to the energy stored within it.

**Figure** 1 : http://www.aircommandrockets.com/rocket_launcher.htm

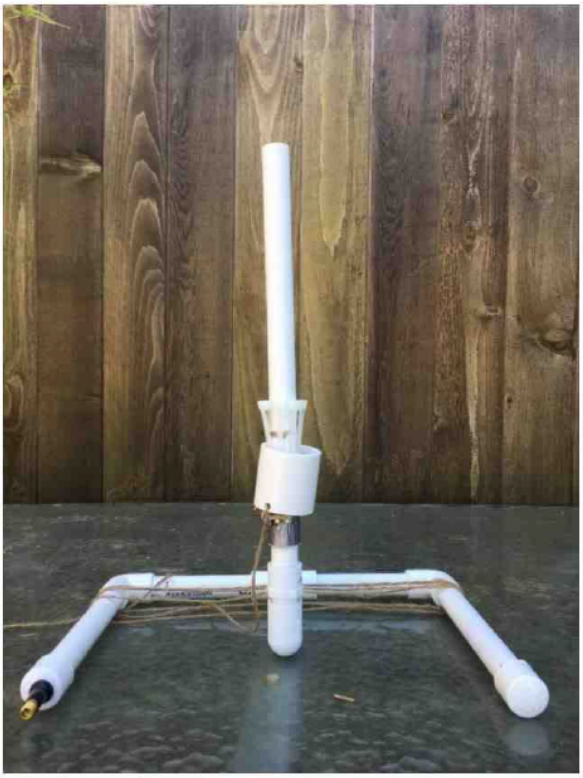


**Figure 2** : Finished water rocket launcher

The rocket was made from two 2 litre polyethylene terephthalate bottles (**Figure 3**). One was used to as the actual body of the rocket while the other was used for the nose cone and the tail fin cylinder, both of which were attached with electrical tape due to its lightness and strong adhesive. Tail fins made of corrugated plastic were attached to the tail fin cylinder to straighten the rocket's trajectory. Finally, a Jolly Logic Altimeter 2 was added inside of the nose cone, in order to measure different components of the rocket's flight. Holes were added to the nose cone to allow air to escape the previously airtight nose cone, as the altimeter uses atmospheric pressure to detect the altitude.

**Figure 3** : Water rocket used in the investigation

# Methodology

For safety and ease, a nearby soccer field was chosen to launch the rocket. Despite purposely choosing a day with low wind speeds I knew it was inevitable and a large field would be good if the rocket were to land far away. The rocket was filled with an explicit amount of water, starting at 200mL. The rocket was then attached to the launching mechanism and the collar is pulled over the rocket's nozzle. The collar and cable ties pushed the nozzle into the bulged part of the PVC creating an airtight seal. The rocket was then pumped full of pressurized air until it was at 40psi. 40psi was chosen as it's a common pressure used in most water rocket competitions as well as the fact that it is a low enough pressure that the rocket does not risk exploding. The string was then pulled a string, which pulled the collar down. This released the rocket and allows it to begin its flight **(Figure 4)**. I did this for all 18 flights. I collected data for 6 different three trials for each water volume to ensure the quality of data. The altimeter recorded peak altitude(apogee), top speed, thrust time, peak acceleration, average acceleration, coast to apogee, the speed of descent, and flight duration.

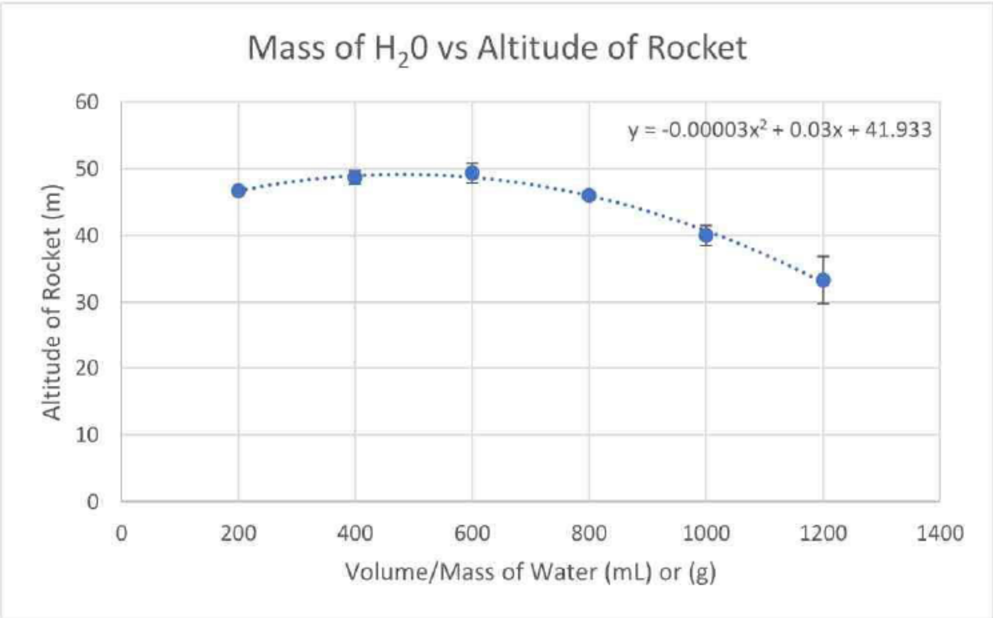
**Figure 4** : Water rocket just after launch

## Experimental Data

The data was collected directly from the Jolly Logic Altimeter. This altimeter is known as being one of the most precise altimeters on the market, with an uncertainty of the smallest digit or smaller. Most values are averaged over three trials. Some data was nonsensical and was omitted, this was attributed to flaws in the altimeter.

| | Trial 1 | Trial 2 | Trial 3 | | | | | |
|---|---|---|---|---|---|---|---|---|
| Vol. (mL) | Apogee (m) | | | Avg. Apogee | | Uncert. | | |
| 200 | 47 | 47 | 46 | 47 | | 0.5 | | |
| 400 | 50 | 48 | 48 | 49 | | 1.0 | | |
| 600 | 51 | 49 | 48 | 49 | | 1.5 | | |
| 800 | 46 | 46 | 46 | 46 | | 0.0 | | |
| 1000 | 41 | 41 | 38 | 40 | | 1.5 | | |
| 1200 | 33 | 37 | 30 | 33 | | 3.5 | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

The following graph can be made for the Volume vs. Altitude of the rocket:



The equation of the curve this line makes is $-0.00003x^2+0.03x+41.933$, which was generated by Excel. Using a Ti-84 graphing calculator the maximum of this graph can be found. The vertex of the graph is (500, 49.433) which means 500mL of water is the optimum volume of water for my water rocket, and if filled to that volume it would travel approximately 49.5 meters.

## Theoretical Data

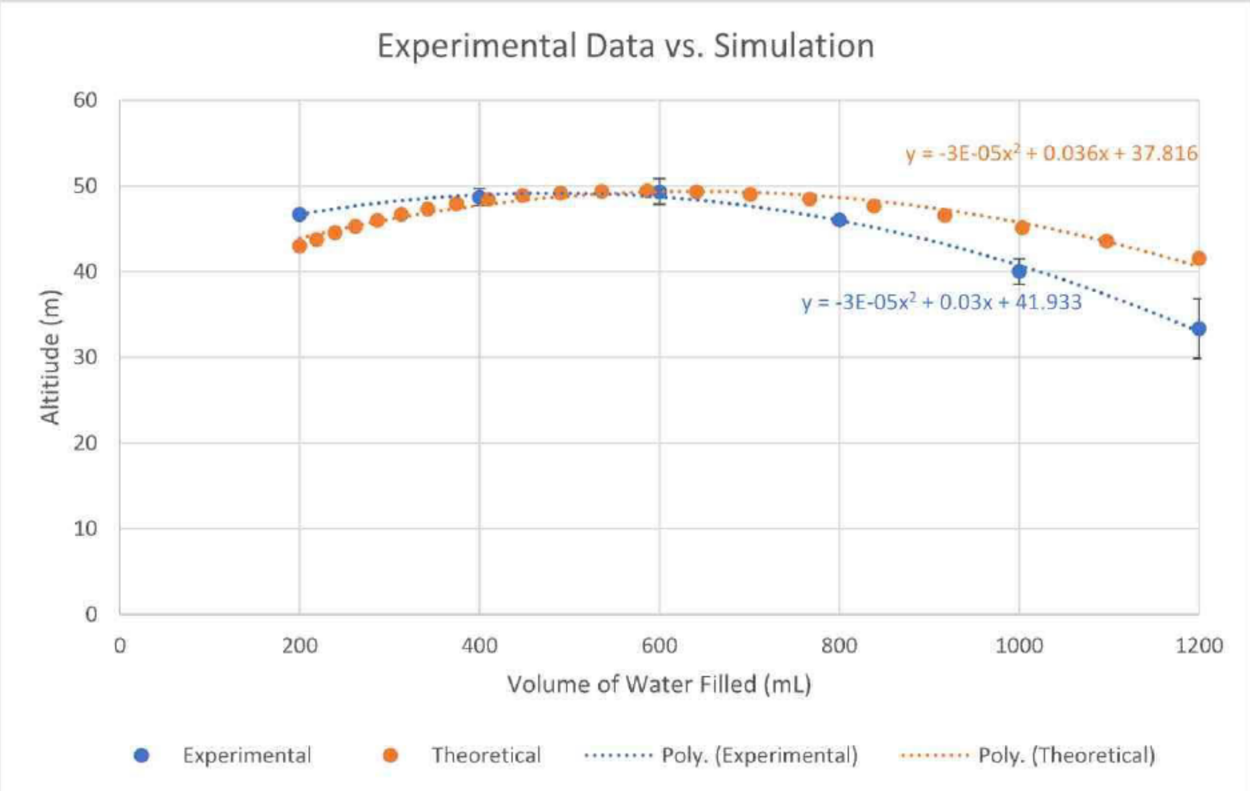
Now compare this data can be compared with an online water rocket simulator. This simulator is used to optimize certain rocket parameters. In the case of this investigation, the parameter that is being optimized is the water volume. Some parameters required for the simulation were unknown, most importantly the drag coefficient. The drag coefficient was estimated by comparing the rocket with shapes with known drag coefficients. This gave me a very rough estimate of the drag coefficient, which was estimated as 0.23. In order to accurately measure the drag coefficient of the water rocket used in this investigation a wind tunnel would be necessary. This is not practical for an investigation of this size nor is it crucial enough to justify. It is also worth noting that the drag coefficient does not change the water volume when the maximum altitude is achieved, it only changes the data systematically up and down the y axis. Because the drag coefficient doesn't change the optimum water to air ratio a rough estimate is justified.

| | |
|---|---|
| **Fuel Capacity** | 2000 mL |
| **Diameter** | 110 mm |
| **Water Fill** | 200 mL |
| **Launch Pressure** | 40 PSI |
| **Nozzle diameter** | 22 mm |
| **Nozzle viscous losses** | 0.20 |
| **Dry mass** | 120 grams |
| **Coefficient of drag** | 0.23 |
| **Launch Tube length** | 225.0 mm |
| **Launch Tube diameter** | 22 mm |

The raw data the simulation gave is the following:

| Volume | Apogee (metres) |
|---|---|
| 200 | 42.97 |
| 218.74 | 43.74 |
| 239.25 | 44.51 |
| 261.67 | 45.26 |
| 286.19 | 45.98 |
| 313.02 | 46.67 |
| 342.35 | 47.31 |
| 374.44 | 47.9 |
| 409.53 | 48.41 |
| 447.92 | 48.84 |
| 489.9 | 49.16 |
| 535.81 | 49.36 |
| 586.03 | 49.42 |
| 640.96 | 49.31 |
| 701.03 | 49 |
| 766.73 | 48.46 |
| 838.59 | 47.66 |
| 917.19 | 46.55 |
| 1003.15 | 45.11 |
| 1097.17 | 43.55 |
| 1200 | 41.51 |

The data set the simulation gave was graphed on the axis as the theoretical data:




## Error

The maximum altitude in the simulation takes place at 600mL. We can compare this with our experimental value using the percent error formula:

$$\text{Percent Error} = \frac{V_{observed} - V_{true}}{V_{true}}$$

$$= \frac{500 - 600}{600}$$

$$= \frac{-100}{600}$$

$$= -16.666666666667\%$$

$$= 16.666666666667\% \text{ error}$$

Because of the many factors that affect a water rockets flight, and several of those factors being almost uncontrollable I believe this error is reasonable. I believe the main cause of error to be weather cocking. This occurs when the rocket turns into the wind after liftoff. This is caused by the wind exerting a sideways force on the rocket, making its trajectory a parabola rather than a straight line. This causes the rocket to achieve a lower apogee. Due to varying amounts of wind during each flight, the corresponding altitude could vary significantly. To remedy this, I took 3 trials of each flight and averaged the altitude. For the most part the uncertainty was relatively low. It did seem like as more water was added the variation in each trial became greater however 800mL was an exception to this as all three trials gave the same value for attitude, which was 46m. If all three flights achieved the same altitudes this would mean that they experienced the same amount of weather cocking.

The horizontal force the wind exerts on the rocket causes it to rotate around its center of gravity. This makes the water flow at an angle to the horizontal. This angle can be calculated using the equation

**tan b = V / w.**

where b is the angle of the flow to the horizontal, V is the velocity of the rocket, and w is the velocity of the wind. The wind speed can be calculated from passed weather supports in my city, North Vancouver. According to *vancouver.weatherstats.ca* the wind speed on February 24th, at 1:00 was 21km/h. I conducted all the flights between 12:30 and 2:00. Although there was some variation in wind speeds between those times the difference is small enough that it can be ignored. For this example, I will use the data I collected from a single trial at 800mL. In this trial the maximum altitude was 46m and the maximum velocity was 48m/s.

$\tan b = 48/21$

$\tan^{-1}(48/21)=66.37°$ to the horizontal.

The number of meters in altitude lost can be modeled by the equation

**H = A * (1 - sin b)**

Where H is the lost altitude, and A is the maximum altitude.

$H = 46 * (1-\sin(66.37))$

H= 3.857 or about 4 meters.

4 meters is a significant amount of lost altitude and because each flight had a different velocity and varying wind speeds this could create a large amount of error.

Another source of error comes from the unevenness of the ground. If the ground, I launched my rocket on was not perfectly horizontal this could have further greatened the angle of the water flow. This of course would have contributed to even more lost altitude. This however is an example of systematic error because every launch would have been affected by this evenly.

Another possible source of error is the large uncertainty present in the foot pump. The meter on the foot pump was not very precise and therefore there may be small variations in initial pressure across different trials. This could be remedied by using an electronic pressure sensor; however this was too expensive for my budget in this investigation.

## Improvements and Extensions

This investigation could be improved by changing the environment it takes place in. Wind greatly affects the flight of a water rocket and an ideal environment would be one with no wind. Either a day with very calm weather, which is fairly rare in Vancouver, or an indoor launch site would work. This would eliminate or reduce the effects of weather cocking dramatically causing the data to be more accurate and precise. Ensuring a perfectly level launch pad would also eliminate some systematic error caused by any unevenness in the grass. This could be achieved using a regular bubble level and would be a very easy way to make the experiment more accurate.

When it comes to the actual apparatus, adding more mass to the nose cone of the rocket would make its flight more stable. The instability of the rockets flight would cause some random error on top of that of the weather cocking. Instability in the rockets flight was not determined to be a major qualitative issue within the experiment but this would minimize it even further. The launching apparatus good but could be improved in several ways. To start, the bulge in the pipe plugged the rocket well but very small amounts of water did escape on some flights. This could be improved by using an O-ring plug rather than a small bulge in the pipe. An automatic filling system within the launcher would also reduce leakage as well as increase the ease of the experiment. This would pair well with a compressed air tank rather than a pump. The compressed air would push the water into the rocket and would force the compressed air into the rocket at a much quicker rate.

One interesting extension would be to see if the optimum water to air ratio stays constant with multiple stage rockets (rockets that have multiple bodies, with each containing its own water). These rockets would have the same challenge of balancing water and air, but of course, because the rocket contains multiple rockets much more air and water could be added. Another interesting extension to this investigation would be to see if the optimum reactive mass to pressurized gas ratio would stay constant for reactive masses of different densities. Using a range of liquids with different densities would not only show how the density affects the optimum reactive mass to pressurized gas ratio but also produce a relationship between density and apogee.

## Conclusion

The data from the experiment shows the ideal volume of water being 500mL, which takes up 1/4$^{th}$ of the rocket's capacity. This does not match the original hypothesis of 1/3$^{rd}$, which is often the standard for competitions. The simulation that the experimental data was compared to comes much closer to the value hypothesized, showing an optimum water amount at 30% full (very close to the hypothetical value of 33.3%).

The theoretical value of 30% comes from the way the water rocket works. If the weight of the rocket was unimportant the ideal fill rate would be around 50% however weight is important. As you fill the rocket with more water it takes longer for it to be forced out by the compressed air. This means that the rocket will have less acceleration at the beginning of the flight due to the added weight of the water. This initial acceleration is formative to the maximum altitude the rocket will reach. The simulator I used took this into account, giving a theoretical value around 30%. This would also mean that the ideal water percentage is dependent on pressure, as at different pressures the time in which the water is forced out of the rocket would vary.

It would be nice to be able to create a mathematical model of the flight of the rocket to complement the experimental and theoretical models, but the mathematics involved were too difficult at this point in my academic career. The following section shows a simplified approximation of some of the values I would have hoped to calculate.

## Further Calculations

Using a range of different equations many other components of the rockets flight can be calculated. For many of these equations several variables are needed. These equations were derived by a technology company called SECME, as the derivations are above my current ability level. The first vital unknown that must be found is the mass flow rate of the water or $\dot{m}$. The water flow rate can be found using the formula $\dot{m} = A \times cd \times \sqrt{(2\rho\Delta P)}$ where A is the area of the nozzle in $m^2$, $C_d$ is the discharge coefficient of the water leaving the nozzle, which was found experimentally by SECME to be approximately 0.98 (SECME, 2013), ρ is the density of water in $kg/m^3$, and ΔP is the average pressure acting on the water. ΔP can be calculated using the formula $\Delta P= (P_i\ (1+V_i/V_f))/2$ where $P_i$ equals initial pressure in $N/m^2$, $V_i$ equals initial volume of air which is 2L-water amount, which we'll take as 0.8L, ad $V_f$ equals final volume of air, which is 2L.

$A=\pi r^2$

$A=\pi(0.022/2)^2$

$A=0.00038m^2$

$C_d= 0.98$

$\rho= 997kg/m^3$

$\Delta P= (P_i\ (1+V_i/V_f))/2$

$\Delta P=(40psi(1+1.2m^3/2m^3))/2$

$\Delta P=32psi$

$\Delta P= 220632\ N/m^2$

$\dot{m} = A \times cd \times \sqrt{(2\rho\Delta P)}$

$\dot{m} = (0.00038m^2)*0.98*(2*(997kg/m^3)*(220632N/m^2))\text{^}(0.5)$

$\dot{m}= 7.81kg/s$

Now that we have the mass flow rate of the water the velocity of the exiting water can be calculated using the equation $V = \dot{m} / \rho A$. With that we can then calculate the thrust of the rocket using the equation $F_t = \dot{m} \times V$.

$V = \dot{m} / \rho A$

$V= (7.81kg/s)/(998kg/m^3*0.00038m^2)$

$V=20.59m/s$

$F_t = \dot{m} \times V$

$F_t = 7.81kg/s*20.59m/s$

$F_t = 160.8N$

We can then plug the value for V into the ideal rocket equation, which is $\Delta u =V \ln(M)$ where Δu is the change in the rocket's velocity, V is the velocity of the exiting water (exhaust velocity), and M is the propellant mass ratio.

$\Delta u =(20.59m/s)*\ln[(0.12kg+0.8kg)/0.12kg]$

$\Delta u =41.94m/s$

With this information the net force of the rocket can be found. The net force is equal to $F_t$-$F_g$-$F_d$ where $F_t$ is force thrust, $F_g$ is force gravity, and $F_d$ is the drag force. $F_d$ can be found using the formula $F_d=0.5(\rho V^2)C_d*A$, where ρ is the density of air, V is the velocity, $C_d$ is the drag coefficient, and A is the cross-sectional area of the rocket in $m^2$.

$F_d=0.5(\rho V^2)C_d*A$

$F_d=0.5(1.225kg/m3*(41.94m/s)^2)0.23*0.00809$

$F_d=2N$

$F_{net}=160.8-((0.12+0.8)(9.8))-2$

$F_{net}=149.784N$

Using $F_{net}=ma$ we can find the acceleration of the rocket.

$a= F_{net}/m$

$a=149.784/(0.12+0.8)$

$a=162.8m/s^2$

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